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ANALYSIS PARETO–NASH–STACKELBERG SOLUTIONS IN SUGAR MARKET PRICING

LOZAN VICTORIA
Department Information Technology and Information Management
Academy of Economic Studies of Moldova
Chisinau, Republic of Moldova
e-mail: lozan.victoria@ase.md
ORCID ID: 0000-0002-5869-8515

Abstract: Within the oligopolistic market, the decisions made by each company regarding price, production or marketing strategy directly influence the reactions and performances of competitors. In such an environment, characterized by a small number of participants and pronounced interdependence, producers frequently face complex decision-making problems, the solution of which directly determines the level of profit obtained. One of the most important strategic decisions is pricing, especially for companies that offer similar products, such as sugar, where competition is intense and differentiation is low.

The paper analyzes the strategic behavior of local sugar producers, considered as two main players, using the Bertrand model with limited capacity (Edgeworth, 1889) and discretized Bertrand model. These models provide a relevant framework for situations where companies compete on price but simultaneously face constraints related to production capacity, which complicates market equilibrium and a framework for equal production capacities that individually cover demand. In the proposed analysis, two possible pricing strategies are considered for each producer, and the results are evaluated in terms of profit obtained for each combination of decisions. In addition to the Nash and Stackelberg solutions in pure strategies, the solutions in mixed strategies are also determined by applying established algorithms that allow the identification of the set of Nash equilibria in mixed strategies (Ungureanu, 2017), the set of Stackelberg equilibria in mixed strategies (Lozan and Ungureanu, 2010), as well as the set of Pareto-Nash solutions in mixed strategies (Lozan and Ungureanu, 2012a) and the set of Pareto-Stackelberg equilibria in mixed strategies (Lozan and Ungureanu, 2012b). This approach provides a more detailed picture of how strategic interaction between sugar producers can lead to efficient outcomes for the entire market.

Keywords: mixed-strategy, graph of best response mapping, set of Stackelberg equilibria, set of Nash equilibria, set of Pareto-Nash equilibria, set of Pareto-Stackelberg equilibria
JEL Classification: C02, C61, C62, C65, C72, C79

1 Introduction

In oligopolistic markets, pricing strategy is an essential component of competitive policy. Setting the price level determines both the dynamics of demand and the distribution of market shares among participating firms. In the context of strategic interdependence, the decision of one firm directly influences the results of the other, which requires analysis using game theory models. In the classical form of Bertrand competition (Bertrand, 1883), firms can choose a continuous price in the interval $[c, \infty)$ (c – marginal cost), and the equilibrium reduces to the paradoxical situation of price equal to marginal cost and zero profit. Therefore, in order to reflect a different strategic dynamic than that in the classical Bertrand model and to perform a more realistic analysis of the interactions between producers, the discretized Bertrand model can be examined, used to describe markets in which prices are not completely flexible, but determined by conventional thresholds, trade policies or institutional constraints. The Bertrand model with limited capacities (Edgeworth, 1889) describes competition in terms of price choice for homogeneous goods, but they are constrained by the limits of their own production capacity. Determining the final outcome of the interaction between firms is relevant by identifying Nash (Nash, 1951), Stackelberg (Stackelberg, 1934), Pareto-Nash-Stackelberg (Lozan and Ungureanu, 2011) equilibria, both in pure and mixed strategies. The importance of this approach lies in the ability to evaluate the conditions of coordination and conflict in the market and to estimate possible levels of profit. Thus, research on pricing strategies within duopolies or oligopolies provides theoretical and applied support for economic decision-making and for the substantiation of competition policies.

2 The sugar market in the Republic of Moldova

The general model of price competition can be customized to analyze the interactions between sugar producers in the Republic of Moldova. The sugar market in the Republic of Moldova is dominated by a small number of large producers. Prices have fluctuated significantly, influenced by domestic production, imports and regulations, and the sector has shown a downward trend, highlighted by the reduction in the volume of processed beet and, implicitly, sugar production. Although Moldovan producers can export sugar, especially under preferential quotas to the European Union, export volumes remain limited, and in recent years the Republic of Moldova has become, to a large extent, a net importer of sugar, due to the decrease in domestic production and the reduced capacities of local factories. If in favorable years domestic production largely covered national consumption, currently it varies between 60 and 100 thousand tons annually, while domestic demand is approximately 75–80 thousand tons (AgroExpert, 2024). For example, in 2024, about 293 thousand tons of sugar beet were harvested (BNS, 2025), which allowed obtaining approximately 32 thousand tons of sugar, an insufficient level to satisfy consumption, the deficit being covered by imports and the use of existing stocks, estimated at 18 thousand tons (Știri.md, 2024). Figure 1 shows the sugar beet harvests for the period 2021-2024.

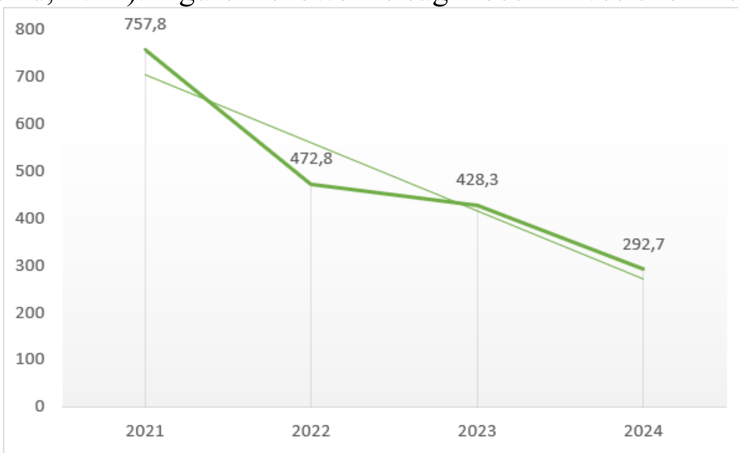

Figure 1. Sugar beet production for 2021-2024 in thousand tons.
Source: Created by the author based on data from (BNS, 2025).

These fluctuations are determined by both climatic factors and the production capacities of domestic factories, which generates structural instability in the market and emphasizes the importance of pricing strategies and trade policies in maintaining the balance between supply and demand.

The paper aims to investigate the scenario in which two sugar factories, called factory A and factory B, have equal capacities, sufficient to fully cover the fixed demand of 6,500 tons per month, and adopt two alternative price levels: a high (H) and a low (L). The analysis aims to maximize profit, subsequently introducing a second profit function, associated with market share, which is subject to the same maximization objective. In this framework, the Pareto–Nash–Stackelberg solutions (Lozan and Ungureanu, 2011) are determined in both pure and mixed strategies. Another scenario analyzed is the one in which the individual production capacity is not sufficient to cover the total demand. The results obtained provide a rigorous perspective on how production capacity, price level and market structure influence the strategic behavior of the factories.

3 The strategic behavior of factories with equal production capacities

We analyze the scenario in which two sugar factories (A and B), with equal production capacities, can individually satisfy the entire fixed monthly demand of Q=6500 t/month. The marginal cost of production is c=15000 lei/ton. Each factory has two alternative pricing strategies: a high level (H=18000 lei/ton) and a low level (L=16000 lei/ton). The factory that charges the lower price covers the entire demand, while the other one sells nothing. If both charge the same price, the demand is distributed symmetrically, respectively $\boldsymbol{q_A = q_B = {}^{Q}/_{2} = 3250\ tons}$. The objective of each factory (player) is to maximize the profit. The profit is expressed through the relationship $\boldsymbol{\pi_i = (p_i - c)q_i}$, where $\boldsymbol{i \in \{A, B\}}$, $\boldsymbol{p_i}$ represents the price charged by the factory *i*, *c* is the marginal cost, and $\boldsymbol{q_i}$ – sold quantity. The Table 1 presents the profit values for the possible situations.

Table 1. The possible situations and the profit obtained by each factory.

B / A	H	L
H	9750000, 9750000	0, 6500000
L	6500000, 0	3250000, 3250000

Source: Created by the author based on problem data.

In the Table 1 are presented on horizontally the pricing strategies of player A, and on vertically those of player B. Each cell contains the profits in lei obtained by A and B in the same situation, separated by a comma.

The Nash equilibrium in pure strategies is (H, H) and (L, L) with the payoffs (9750000, 9750000) and (3250000, 3250000), respectively. When player A chooses strategy H, it is optimal for player B to adopt strategy H, and vice versa. This generates a stable and profitable situation for both factories, since neither has an incentive to unilaterally reduce the price, a possible reduction would secure the entire market, but would lead to a decrease in total profit. Similarly, if player A chooses strategy L, player B's rational response is also L, since any unilateral deviation to H would lead to a loss of sales and a zero profit. The Nash solution set in mixed strategies (Ungureanu, 2017) – a probabilistic distribution on the set of pure strategies, whereby a player chooses each pure strategy with a certain probability – is $\{(0,0), ({}^{1}/_{2}, {}^{1}/_{2}), (1,1)\}$, represented graphically in figure 2.

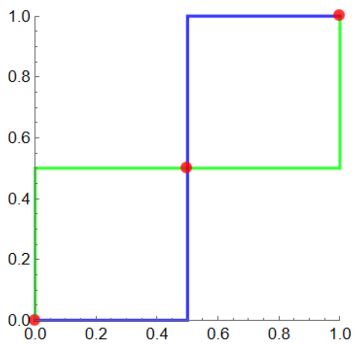

Figure 2. Players' best-response mappings and mixed-strategy Nash equilibria.

Source: Created by the author based on the problem data.

The solution in mixed strategies $({}^{1}/_{2}, {}^{1}/_{2})$ assumes that each player chooses strategy H with probability 1/2 and strategy L with probability 1/2. At the market level, there is no deterministic outcome, but a situation of probabilistic equilibrium. No player has an incentive to unilaterally deviate from this distribution, since any deviation would lead to a lower expected gain. The equilibrium expresses a strategic uncertainty, each player randomly selects between the high and low price, which can be interpreted as a form of average stability, in which the profit is maintained at a constant expected level, and the risk of loss is mitigated.

The Stackelberg solution in pure and mixed strategies (Lozan and Ungureanu, 2010) coincide - the situation (H, H) with the profits (9750000, 9750000). Player A is considered to occupy the position of leader, although the role could be assumed by any of the players. The leader makes the decision first, and the follower reacts optimally. The high price becomes the natural coordination, with both players obtaining high profits and sharing the demand. The equilibrium highlights the advantage of the first player and suggests an implicit strategic alignment induced by the sequential structure of the game.

Next for each player is introduced a second payoff function corresponding to the market share held, expressed in monetary terms. The objective of each producer is to increase market share while maximizing profit.

Taking into account the initial data:
the reference price is 17000 lei/ton;

market value at this price: $\boldsymbol{R_{ref}}$ =17000 lei/ton × 6500 tons = 110500000 lei,

monetary value of market share: $V_i = {}^{q_i}/_{Q}\, R_{ref}$, where $i \in \{A, B\}$, Q - demand, and q_i – sold quantity.

Table 1 is modified, the game from table 2 is obtained.

Table 2. The possible situations and payoffs for players A and B depending on profit and monetary value of market share.

B A	H		L	
H	(9750000, 55250000)	(9750000, 55250000)	(0, 0)	(6500000, 110500000)
L	(6500000, 110500000)	(0, 0)	(3250000, 55250000)	(3250000, 55250000)

Source: Created by the author based on problem data.

Table 2 shows horizontally the gains obtained by factory A, and vertically - the gains obtained by factory B. The first pair of monetary values in the table cells represents the utilities obtained by factory A in the respective situation, and the second pair the utilities obtained by factory B in the same situation.

The Pareto-Nash solution in pure strategies (Lozan and Ungureanu, 2011) is (H, H) and (L, L). For both players it is rational to choose the same pricing strategy as the competitor, thus ensuring their profit and sharing the market equally.

The determination of the set of Pareto-Nash solutions in mixed strategies, taking into account the data in Table 2, is performed using the algorithm described in (Lozan and Ungureanu, 2012a) for dyadic bicriteria games. The results of the calculations are presented in Figure 3. Figure 3a represents the graph of efficient responses application of factory A, Figure 3b – the graph of efficient responses application of factory B, Figure 3c – the Pareto-Nash solution in mixed strategies (intersection of the players' graphs.)

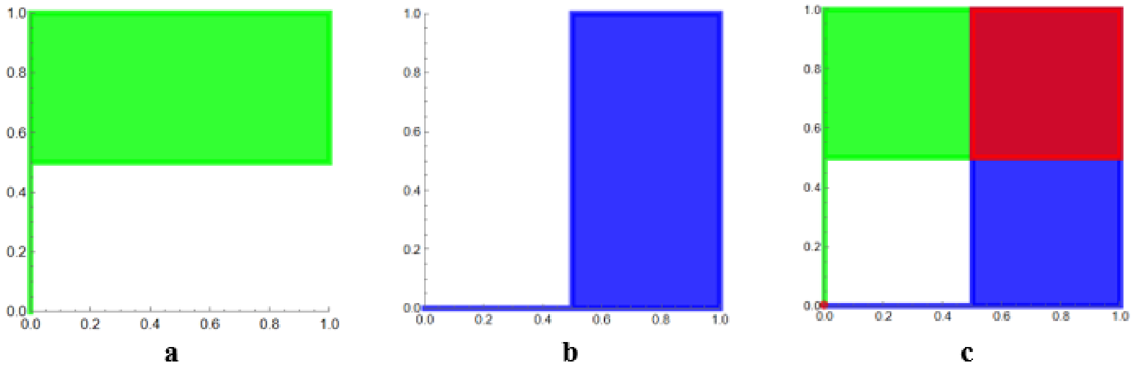

Figure 3. Graphs of efficient responses applications of factories and the Pareto-Nash solution set.

Source: Created by the author in Wolfram Mathematica.

The set of Pareto-Nash solutions in mixed strategies is $\{(0,0)\} \cup \{(1/2, 1/2), (1, 1/2), (1,1), (1/2, 1), (1/2, 1/2)\}$.

The solution indicates that several equilibrium states can coexist in the market, from competition based on low prices to coordination at high price levels or strategic combinations, which highlights the multiple and uncertain nature of competitive equilibrium.

In the case of determining the set of Pareto-Stackelberg solutions (Lozan and Ungureanu, 2011), assuming that A is the leader, in pure strategies the situation (H, H) is obtained. As in the case of the game with a single payoff function, the leader dictates the price, it is not rational for factory B to choose the low price, it will win the market, but the profit will be lower than in the case of the high price.

The algorithm in (Lozan and Ungureanu, 2012b) is used to determine the set of Pareto-Stackelberg equilibria in mixed strategies. Taking into account the data in table 2, the results presented in figure 4 are obtained.

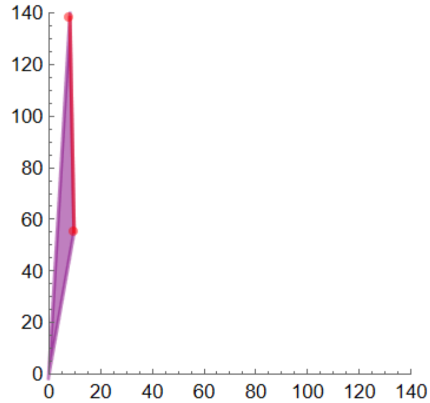

Figure 4. Image of the graph of the application of efficient reactions of factory B in the criteria space of factory A.

Source: Made by the author in Wolfram Mathematica.

The set of Pareto-Stackelberg solutions in mixed strategies is the segment $[(1/2, 1), (1,1)]$. The result highlights that, within the sequential game, the Pareto-Stackelberg equilibrium gives the leader a margin of strategic flexibility in choosing the probability associated with the high price, without affecting the Pareto efficiency conditions. This situation reflects a strong tendency of coordination towards high price levels, which reduces the intensity of competition and contributes to maintaining the stability of profits in the market.

4 The strategic behavior of factories with different capacities

The scenario analyzed in the previous paragraph is modified by establishing different production capacities of the sugar factories (factory A – 3500 t/month, factory B – 4000 t/month), but together they satisfy a fixed monthly demand of Q=6500 t/month. The marginal cost of production remains c=15000 lei/ton. Each factory has the same alternative pricing strategies: a high level (H=18000 lei/ton) and a low level (L=16000 lei/ton). The factory that adopts the lower price strategy sells all available stock, the other covers the remaining demand. If both practice the same price, demand is distributed proportionally to capacity, respectively $\boldsymbol{q_A = 3033\ tons}$, $\boldsymbol{q_B = 3467\ tons}$. The objective of each factory is to maximize profit. Table 3 presents the profit values for possible situations.

Table 3. The possible situations and the profit obtained by each factory in the scenario with different production capacities

B / A	H	L
H	9099000, 10401000	7500000, 4000000
L	3500000, 9000000	3033000, 3467000

Source: Created by the author based on problem data.

The Table 3 presents horizontally the profits in lei obtained by A and vertically - the profit obtained by B in the same situation, separated by a comma.

Based on the data in Table 3, it is found that the high-price strategy is a dominant strategy for both players. The situation (H, H) maximizes aggregate profit, and neither player has the motivation to unilaterally deviate towards a low price, since capacity constraints limit the quantity sold and reduce the profit obtained. In the respective scenario, the Nash solution and the Stackelberg solution, both in pure and mixed strategies, coincide - (H, H), with a profit of (9099000, 10401000).

If the second profit function for factories is introduced, which reflects the market share, then the utilities in Table 4 are obtained, horizontally – the utilities of factory A, and vertically – the utilities of factory B.

Table 4. The possible situations and gains for players A and B depending on profit and monetary value of market share in the scenario with different production capacities

B / A	H		L	
H	(9099000, 51561000)	(10401000, 58939000)	(7500000, 42500000)	(4000000, 68000000)
L	(3500000, 59500000)	(9000000, 51000000)	(3033000, 51561000)	(3497000, 58939000)

Source: Created by the author based on problem data.

Analyzing the data in Table 4 and based on (Lozan and Ungureanu, 2011), all possible situations in pure strategies are Pareto-Nash solutions. The set of Pareto-Nash equilibria in mixed strategies, determined by applying the same algorithm as in the previous scenario, coincides with the square $[0,1] \times [0,1]$. Any strategy leads to a state in which profits and market shares cannot be simultaneously improved for both actors, a balanced but strategically unidentifiable market. Players can make the choice for any possible situation.

The set of Pareto-Stackelberg equilibria in pure strategies, if the player with the higher production capacity is considered the leader, consists of all possible situations in Table 4. The solution is the same when the player with the lower production capacity is in the leading position. In the case of determining the set of Pareto–Stackelberg equilibria in mixed strategies, the same reasoning and algorithms are applied as in the previous scenario. The segment [(1, 0), (1, 1)] is obtained. The obtained solution indicates that the player in the leading position has full strategic freedom in choosing the probability associated with the high-price strategy, while the successor player is constrained to opt with certainty for the same strategy – high price. This reflects an asymmetry of roles, the leader benefits from flexibility and coordination power, and the deterministic reaction of the successor ensures the stability of the results. From an economic point of view, the solutions segment suggests a robust tendency to maintain high prices in the market, which reduces competitive intensity and favors the maximization of aggregate profits, approaching an implicit coordination mechanism generated by the sequential game structure.

5 Conclusion

The correct choice of a pricing strategy ensures the increase of profits and the expansion of market share. The comparative study of the two analyzed scenarios – with equal and, respectively, different production capacities – highlights the role of the capacity structure on the strategic behavior of players in the market. In the situation of equal capacities, the results show that the competitive interaction generates multiple types of equilibrium, including mixed strategies, which reflects a strategic flexibility and the possibility of adjusting decisions in a way that maximizes collective profits, without the need for an explicit agreement. In contrast, when capacities differ, the high-price strategy emerges as dominant for both players, which reduces uncertainty and leads to the stabilization of the market around a higher price level.

The structure of production capacities directly influences the nature of competitive equilibria and, implicitly, market dynamics. The Pareto-Nash-Stackelberg results obtained contribute to a deeper understanding of how firms adapt their strategic decisions in oligopolistic competition contexts and provide useful benchmarks for the substantiation of managerial strategies and regulatory policies.

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